

# Atwood Machine

---

## Equipment

- Computer, PASCO Interface
- Table Clamp
- Double pulley apparatus (one smart pulley)
- Smart Pulley Data Cable
- String
- Two Mass Hangers
- One Mass Set (1×500, 2×200, 1×100, 1×50, 2×20, 1×10, 1×5, 2×2, 1×1)
- Styrofoam Pad

## Objectives

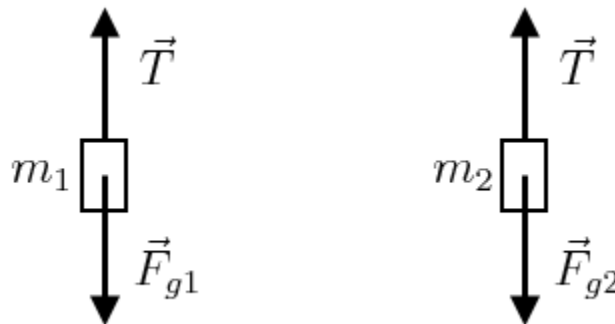
- *To verify Newton's Second Law as applied to an Atwood Machine.*

## Introduction

According to Newton's 2nd Law the acceleration of an object depends on the net applied force and the object's mass.

$$\Sigma \vec{F} = m\vec{a} \quad (1)$$

In an Atwood Machine, there are two objects, each one is a mass hanging from one end of a string. The string is hung over a pulley so that both masses are hanging. The weight of one mass pulls the system in one direction, while the weight of the other mass pulls in the other direction.



**Figure 1.** Individual free body diagrams of each of the hanging masses.

In the above free body diagrams,  $T$  is the tension in the string, assuming that the pulley is massless and frictionless, and the string has no mass and doesn't stretch, the tension  $T$  will be the same throughout the string. The weights are  $F_{g1} = m_1g$  and  $F_{g2} = m_2g$  where  $g$  is the acceleration due to gravity. Taking the convention that up is positive and down is negative, the net force equations for  $m_1$  and  $m_2$  are:

$$\sum F_{1y} = T - m_1g = m_1a_1 \quad (2)$$

$$\sum F_{2y} = T - m_2g = m_2a_2 \quad (3)$$

The key to solving this is to realize that in the system of  $m_1$  and  $m_2$ , one mass accelerates upward while the other accelerates downward. So, for mass  $m_2$ , ***we flip the coordinates*** and then the accelerations of  $m_1$  and  $m_2$  are the same and we can just call them both  $a$ .

$$\sum F_{1y} = T - m_1g = m_1a \quad (4)$$

$$\sum F_{2y} = -T + m_2g = m_2a \quad (5)$$

We can add equations (4) and (5) to obtain:

$$m_2g - m_1g = m_1a + m_2a \quad (6)$$

$$(m_2 - m_1)g = m_{\text{tot}}a \quad (7)$$

Solving for  $a$ , the acceleration of the system of both masses, the theoretical acceleration is  $g$  times the difference in mass divided by the total mass:

$$a = g \left( \frac{(m_2 - m_1)}{m_{\text{tot}}} \right) \quad (8)$$

If  $m_2 > m_1$ , then  $F_{g2}$  is stronger, and the net force on the system accelerates  $m_2$  downward and  $m_1$  upward.

### Treating the Atwood Machine as a System.

Notice that Eq. 7 looks a lot like Newton's 2nd Law (Eq. 1). The "object" is the pair of masses attached to either end of a string. On the right side of Eq. 7, the mass of the system is the total mass of the two hangers. The acceleration of the system is directly proportional to the exerted force and inversely proportional to the mass of the system.

On the left of Eq. 7, we have the net force on the system  $\sum F = (M - m)g$ . The net force acting on the system of both masses is just the difference in weight between two hanging masses. This net force accelerates both hanging masses. The heavier mass is accelerated downward, and the lighter mass is accelerated upward.

## Experimental Plan

Newton's 2nd Law can be verified by setting up the Atwood Machine under various conditions. By carefully selecting the masses, we can graph:

- (1)  $\Sigma F$  vs.  $a$  while keeping the system mass  $\Sigma m$  constant.
- (2)  $a$  vs.  $1/\Sigma m$  while keeping the net force,  $\Sigma F = (M-m)g$ , constant.

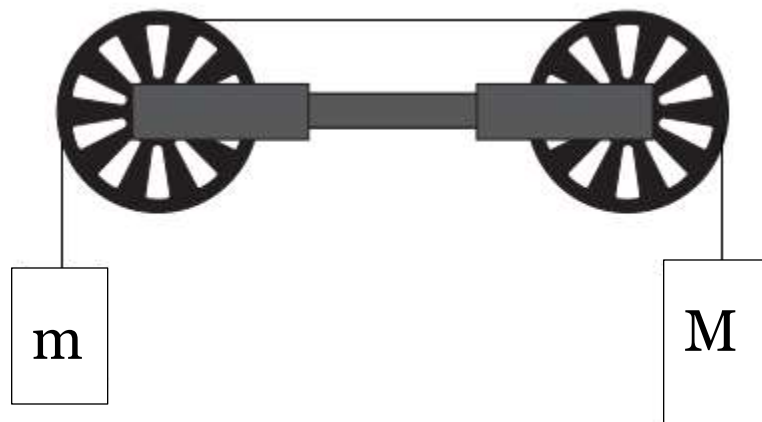
In graph (1), the result should be linear with the slope of the graph equal to the total mass. In graph (2), the result should be linear with the slope of the graph equal to the exerted net force  $(M - m)g$ . If this is true, then Newton's 2nd Law is verified.

## Experimental Setup

We will use the Photogate/pulley system to measure the motion of both masses as one moves upward and the other moves downward. Capstone software will record the changing speed of the masses as they move. The recorded data will be displayed as a velocity vs. time graph, where the slope of the graph is the acceleration of the system. Capstone file "Atwood's Machine" is posted on Blackboard in a corresponding folder and should be open before starting the experiment.

## Hardware Setup

1. Mount the universal clamp to the edge of the table.
2. Insert the connecting rod of the pulley into the clamp.
3. Plug the photogate into digital channel 1 of the interface. You can tell it's working by spinning the pulley. A little red light near the pulley should blink.
4. Use a piece of string just long enough so that when one mass is on the floor/table, the other is a few centimeters below its pulley.
5. Hang mass hangers from each end of the string, and hang the string over the pulleys, as shown in Figure 2.



**Figure 2.** Schematic representation of the Atwood Machine. (Image source: PASCO)

6. Make sure the string isn't rubbing against anything, such as the data cable.

## Part 1: Constant Total Mass

- Place the weights from the mass set on the holders following the pattern below:
  - $M$  (large mass) = 0.36 kg = 360 grams. Holder (50 g) + 200 g + 100 g + 10 g
  - $m$  (small mass) = 0.34 kg = 340 grams. Holder (50 g) + 200 g + 50 g + 20 g + 20 g
  - The total mass should be 0.7 kg = 700 grams.
- Position the masses so that  $M$  is hanging near the top and  $m$  is on or near the Styrofoam pad.
- When you are ready to start, press **Start** in Capstone and release system.
- When  $M$  reaches the bottom, press **Stop**.
- Fill in the columns of the Table 1.
  - Run #: Record the run number from Capstone, in case you should come back to it.
  - $m$ , and  $M$ : The individual masses including the hanger.
  - $a$ : the experimental acceleration recorded by Capstone.
  - $\Sigma F = (M - m)g$ : net force acting on the system.  
DO NOT USE the measured acceleration for this column!
  - $\Sigma m$ : the total mass.

| Run # | $m$ (kg) | $M$ (kg) | $a$ (m/s <sup>2</sup> ) | $\Sigma F$ (N) | $\Sigma m$ (kg) |
|-------|----------|----------|-------------------------|----------------|-----------------|
|       |          |          |                         |                |                 |
|       |          |          |                         |                |                 |
| ...   |          |          |                         |                |                 |

**Table 1.** Example table for recording run parameters, results, and calculations for Part 1.

- Perform 4 more trials. For each trial, move 10 g from  $m$  to  $M$ . (Sometimes you have to move 20 g and move 10 g back.) Use the collected data to fill in Table 1.
- Plot  $\Sigma F$  vs.  $a$  and fit it into linear trendline. Record your results in Table 2. To validate 2<sup>nd</sup> Law of Motion, the slope of the linear trendline should be equal to the total mass of the system in kilograms. Compare the two by calculating % Error =  $\text{ABS}(\text{Slope} - \Sigma \text{Mass}) / \Sigma \text{Mass}$

|                                  |  |
|----------------------------------|--|
| Total Mass (kg)                  |  |
| Slope of $\Sigma F$ vs. $a$ (kg) |  |
| % Error                          |  |

**Table 2.** Experimental and expected total mass.

## Part 2: Constant Net Force

The data runs will always have the same mass difference, hence the same net force.

- Choose the masses for the first run. With our mass sets, this is a good amount:
  - $M$  (large mass) = 0.35 kg = 350 grams: Holder (50 g) + 200 g + 100 g
  - $m$  (small mass) = 0.25 kg = 250 grams: Holder (50 g) + 200 g
- Position the masses so that  $M$  is hanging near the top and  $m$  is on or near the Styrofoam pad.
- When you are ready to start, press **Start** in Capstone and release system.
- When  $M$  reaches the bottom, press **Stop**.
- Fill in the columns of the Table 3.
  - Run #: Record the run number from Capstone, in case you should come back to it.
  - $m$ , and  $M$ : The individual masses including the hanger.
  - $a$ : the experimental acceleration recorded by Capstone.
  - $\Sigma F = (M - m)g$ : net force acting on the system.
  - $\Sigma m$ : the total mass.

| Run # | $m$ (kg) | $M$ (kg) | $1/\Sigma m$ (kg <sup>-1</sup> ) | $a$ (m/s <sup>2</sup> ) | $\Sigma F$ (N) |
|-------|----------|----------|----------------------------------|-------------------------|----------------|
|       |          |          |                                  |                         |                |
|       |          |          |                                  |                         |                |
| ...   |          |          |                                  |                         |                |

**Table 3.** Example table for recording run parameters, results, and calculations for Part 2.

- Perform 4 more trials, adding 10 g to each hanger before each trial (one 10g set comes from smaller weights: 5+2+2+1). Use the collected data to fill in Table 3. (Note: You cannot make  $M=0.39$  kg with  $m=0.29$  kg. Just skip to the next point.)
- Plot  $a$  vs.  $\frac{1}{\Sigma m}$  and fit it into linear trendline. Record your results in Table 4. To validate 2<sup>nd</sup> Law of Motion, the slope of the linear trendline should be equal to the net force acting on the system. Compare the two values by calculating % Error =  $\text{ABS}(\text{Slope} - \Sigma F)/\Sigma F$

|   |  |
|---|--|
| Net Force (N)                             |  |
| Slope of $a$ vs. $\frac{1}{\Sigma m}$ (N) |  |
| % Error                                   |  |

**Table 4.** Experimental and expected net force.

## Requirements for the Report (also consult the rubric):

Save your Excel files through Canvas Group File Exchange

- The **abstract section** must contain the following explanations in paragraph form:
  - How the data was collected and calculated for Table 1
  - How the data from Table 1 was analyzed including interpretation of the trendline and comparison of values in Table 2
  - How the data was collected and calculated for Table 3
  - How the data from Table 3 was analyzed including interpretation of the trendline and comparison of values in Table 4
  - A general statement based on Tables 2 and 4 about how the experimental results support Second Law of Motion ( $\Sigma F = \Sigma ma$ )
- The **data section** must include
  - 4 Tables (labeled and captioned)
  - 2 Graphs (title, axis labels, units, labeled and captioned)
    - $\Sigma F$  vs  $a_{\text{exp}}$
    - $a_{\text{exp}}$  vs  $\frac{1}{\Sigma m}$  (*Inverse Mass,  $kg^{-1}$* )